Power Transmission in Microwave Systems

or

What the Hell is Going on Between Magnetron and Load!

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Contents

1. Introduction

This work was motivated by a well-known, yet rather unexpected phenomenon, sometimes observed in microwave systems consisting of a signal source, a transmission line, and a load: the power carried by a wave travelling toward the load can appear to be greater than the power available from the signal source. Specifically, in high-power industrial applications, the power supplied by a magnetron can be computed from the anode voltage, anode current, and the device efficiency. The power carried by a wave travelling to the load can be measured using a directional coupler. Surprisingly, it has been observed that this power can sometimes significantly exceed the power available from the magnetron! Such outcomes can raise doubts about the accuracy of the measurement devices (couplers) or the validity of the measurement method. This paper aims to:

- Clarify the phenomenon for readers, particularly those who use microwave technology but are not specialists, by providing an explanation of the observed effects.
- Recommend appropriate measurements that will yield meaningful quantities for characterizing system behavior, and outline the accuracy limits of such measurements.

To support this effort, we developed the Windows-based **PowTrans** application (Microwave Power Transmission Calculator). **PowTrans** supplements the theory by enabling a variety of useful simulations, making it a practical tool for real-world assessments.

2. System Overview

The system we will examine in this work consists of three basic blocks (Fig. 1):

- Signal source (magnetron).
- Transmission medium (transmission line, waveguide).
- Load (applicator, working space).

The distributed-network representation of the system is shown in Fig. 2. The source-to-waveguide interface plane is denoted T_S ; the waveguide-to-load interface plane is denoted T_L .

In the following discussion, we assume that the system is linear, the transmission medium is lossless, and the signals are harmonic.

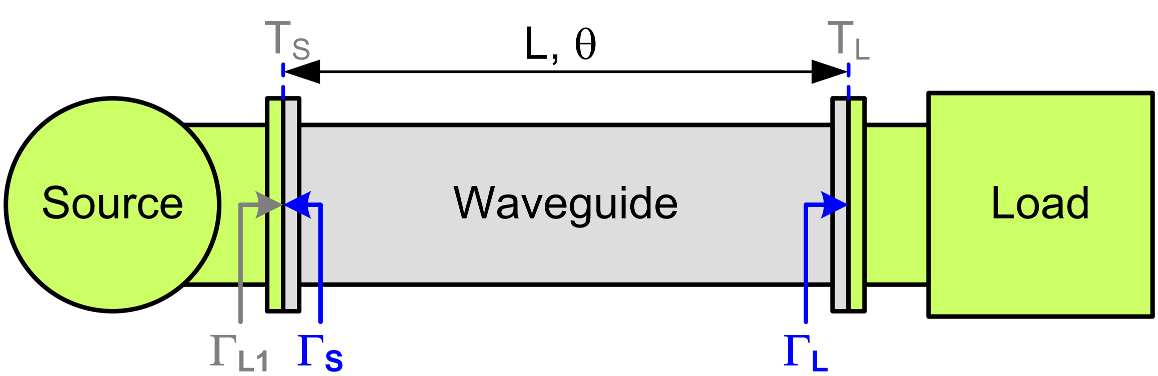


Fig. 1. The three basic blocks of our system of interest.

2.1 Load

We characterize the load (an applicator) either by its complex impedance Z_L or, more generally, by its reflection coefficient

$$\Gamma_L = |\Gamma_L| \exp(j\varphi_L)$$

The two quantities are related via

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1}$$

where Z_0 is an arbitrary real *reference impedance*. For TEM transmission lines (such as coaxial lines), it is meaningful to set Z_0 equal to the characteristic impedance of the transmission line. In waveguide systems, it is common to either set $Z_0 = 1$, or work with *normalized impedances*

$$z = Z/Z_0$$

A load is said to be *matched* when $\Gamma_L = 0$, or, when impedances can be unambiguously defined, when $Z_L = Z_0$. Thus, the notion of a *match* is always relative to a specific transmission line type or reference impedance.

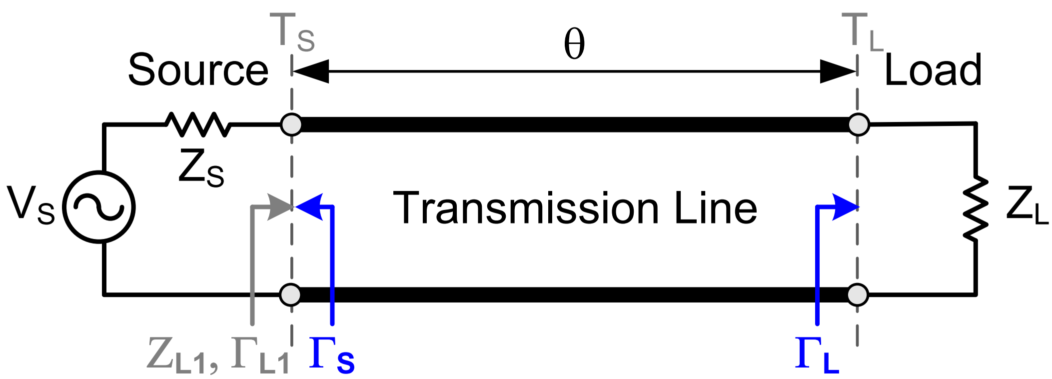


Fig. 2. Network representation of our studied system.

2.2 Transmission Medium

In general, a transmission medium can be represented as an arbitrary linear two-port network connecting a source and a load. Such a network is fully characterized by its four scattering parameters. In most practical applications, the transmission medium is a section of a nearly lossless transmission line or waveguide.

The fundamental characteristics of a transmission line (from which its scattering parameters can be derived) are:

- **Characteristic impedance** Z_0. For waveguides, the definition of Z_0 is ambiguous; it is often convenient to assume $Z_0 = 1$.
- **Electric length** θ (in radians) is defined as

  $$\theta = 2\pi L / \lambda \tag{2}$$

  where L is the physical length of the line and λ is the *guide wavelength*, i.e., the wavelength of the wave propagating along the line. In degrees, this becomes

  $$\theta^{\circ} = \theta \times 180/\pi$$

2.2.1 Wave

A wave travelling in a transmission line in the positive x-direction along the line can be mathematically represented by a *complex wave amplitude*, or *phasor*

$$a(x) = A\exp\left[j\varphi(x)\right] = A\exp\left[j\left(\varphi_0 - \frac{2\pi x}{\lambda}\right)\right] = A\exp\left[j\left(\varphi_0 - \theta\right)\right] \qquad (3)$$

Here A is the real-valued wave amplitude, x is the distance from an arbitrarily chosen initial location, $\varphi_0 = \varphi(0)$ is the initial phase, λ is the guide wavelength, and θ is the electric distance corresponding to the physical distance x.

The amplitude A is chosen such that the power carried by the wave is

$$P = \frac{1}{2}A^2 \qquad (4)$$

This relation highlights that A is a measure of the electric *field strength* (intensity) of the physical wave rather than its power. In the case of TEM waves, A is a measure of the voltage on the line.

Thus, for our purposes, a wave can be sufficiently characterized by the following three measurable quantities:

- Wavelength λ
- Phase angle $\varphi(x)$ at a given distance x
- Mean power P carried by the wave

The power-intensity relation (4) has a very practical consequence, often overlooked in cases when multiple waves interfere: To obtain the resulting power of a combination of multiple coherent waves[1], their complex *amplitudes* must be summed, not their powers.

Thus, in min-max power estimations, we should add or subtract *the square roots* of the individual wave powers, and then take the square of the result. The following example illustrates the difference.

Example: Interference of Two Coherent Waves

Problem: Two waves of the same type and frequency travel in the same direction. One carries a power of $P_1 = 1000$ W, the other $P_2 = 500$ W. The relative phase between them is either unknown or may vary. What are the maximum and minimum possible powers of the resulting wave?

Solution: The maximum power occurs when the two waves are in phase, allowing their amplitudes to add algebraically. Conversely, the minimum power occurs when the waves are 180 degrees out of phase, causing their amplitudes to subtract algebraically. The total power in each case is given by

$$P = \left(\sqrt{P_1} \pm \sqrt{P_2}\right)^2$$

yielding $P_{min} = 85.8$ W, $P_{max} = 2914.2$ W. In contrast, if the powers were added or subtracted directly, one would obtain $P_{min} = 500$ W, $P_{max} = 1500$ W, significantly underestimating the actual range due to neglecting phase coherence.

When a wave travels a distance L in the positive x direction, its phase decreases proportionally to the electric length θ:

$$a_L = a_0 \exp\left(-j2\pi L / \lambda\right) = a_0 \exp\left(-j\theta\right)$$

That is:

[1] Two waves are coherent if they have the same frequency and maintain a fixed phase difference.

$$\varphi_L = \varphi_0 - j\theta$$

If a transmission line with electric length θ is terminated by a load with reflection coefficient Γ_A, then the reflection coefficient seen at the input of the line is

$$\Gamma_L = \Gamma_A \exp(-j2\theta) = \Gamma_A \exp(-j4\pi L/\lambda)$$

2.2.2 Power Balance

A wave travelling in a transmission line toward load is termed *incident wave* or *forward wave*. When such a wave reaches a load with reflection coefficient Γ_L, only a portion of its energy is absorbed in the load. The remaining energy is reflected, generating a *reflected* or *reverse wave* that travels back to the source. Let P_i be power carried by the incident wave. Then the reflected power is

$$P_r = P_i |\Gamma_L|^2 \tag{5}$$

The absorbed (transmitted) power is

$$P_L = P_i - P_r = P_i \left(1 - |\Gamma_L|^2\right) = P_i m_L \tag{6}$$

Here, the factor

$$m_L = 1 - |\Gamma_L|^2 \tag{7}$$

is known as the *load mismatch factor*. It quantifies the fraction of incident power that is successfully delivered to the load.

2.3 Signal Source

In the theory of linear lumped-element networks and networks containing also TEM transmission lines (not waveguides), a signal source can be characterized by two quantities:

- The open-circuit voltage amplitude[2] V_S
- The source internal impedance Z_S

Both V_S and Z_S are generally complex quantities. However, if the system of interest contains only one signal source (as is the case here), we can, without lack of generality, assume V_S is real.

A more general approach, that applies also to systems involving waveguides, is to characterize a signal source by the alternative set of parameters:

- The available power P_{av}
- The source reflection coefficient $\Gamma_S = |\Gamma_S|\exp(j\varphi_S)$

2.3.1 Source Reflection Coefficient

Just as with the load, the source impedance and the source reflection coefficient are related by

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \tag{8}$$

If the source voltage V_S was reduced to zero without altering any other circuit characteristics, the source would behave like a pure impedance Z_S (or reflection coefficient Γ_S), analogous to the load case. Therefore, when a wave reflected from the load reaches the source, its power is partially

[2] Amplitude means the peak value of a harmonic signal.

absorbed within the source and partially re-reflected. This re-reflected wave adds to the resultant forward wave propagating toward the load.

The source is said to be *matched* when $Z_S = Z_0$, or, equivalently, when $\Gamma_S = 0$. In analogy to the load, we define the *source mismatch factor* as

$$m_S = 1 - |\Gamma_S|^2 \quad (9)$$

In practice, a signal source can be a composite device that includes auxiliary components to improve its match. For example, high-power magnetrons are often used in combination with three-port devices known as circulators. A circulator redirects any wave reflected from the load away from the magnetron and into an auxiliary high-power termination, where it is safely absorbed. This protects the source from damage and maintains a low $|\Gamma_S|$, i.e., good source match.

2.3.2 Gross Power

The gross power P_g is defined as the total power drawn from the active element of the source, i.e., the ideal voltage source in Fig. 2. Using the lumped-element equivalent circuit (Fig. 3) with $Z_S = R_S + jX_S$ and $Z_L = R_L + jX_L$, the gross power can be computed as

$$P_g = \frac{1}{2}\mathrm{Re}\left\{V_S I^*\right\} = P_S + P_L$$

where[3]

$$P_S = \frac{1}{2}|V_S|^2 \frac{R_S}{|Z_L + Z_S|^2} = 4P_{av} \frac{R_S^2}{|Z_L + Z_S|^2} \quad (10)$$

is the power dissipated in the source itself (specifically in the real part R_S of the source impedance Z_S), and

$$P_L = \frac{1}{2}|V_S|^2 \frac{R_L}{|Z_L + Z_S|^2} = 4P_{av} \frac{R_L R_S}{|Z_L + Z_S|^2} \quad (11)$$

is the power absorbed by the load. Note that Z_L is the load impedance as seen at the source terminals (denoted Z_{L1} in Fig. 2). The quantity P_{av} refers to the source available power, which will be defined in section Available Power.

These formulas illustrate how the total gross power P_g delivered by the active element is partitioned. A portion P_S is lost internally, often manifesting as heat within the source. The remaining portion P_L is transmitted to the load. The source efficiency, which is the fraction of gross power that is actually delivered to the load, is defined as

$$\eta_S = \frac{P_L}{P_g} = \frac{R_L}{R_L + R_S} \quad (12)$$

[3] The asterisk denotes the complex conjugation.

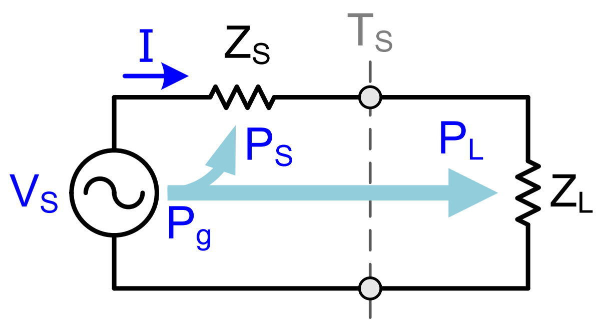


Fig. 3. Lumped-element equivalent circuit of the system of interest.

2.3.3 Available Power

In linear network theory, the available power P_{av} is a commonly used parameter that characterizes a source. It represents the maximum real power that a linear source with internal impedance Z_S can deliver to a passive load[4].

To find the conditions for maximum power transfer, we maximize P_L from Equation (11) with respect to R_L and X_L. This yields the condition

$$Z_L = Z_S^*, \quad i.e., \quad R_L = R_S, \quad X_L = -X_S$$

That is, the load impedance Z_L must equal the complex conjugate of the source impedance. The equivalent condition in terms of reflection coefficients is

$$\Gamma_L = \Gamma_S^*$$

Under these conditions, the available power is given by

$$P_{av} = \frac{1}{8}\frac{V_S^2}{R_S} \qquad (13)$$

where V_S is the *amplitude* (not the effective value) of the open-circuit voltage.

Available power can also be readily defined in waveguide-based networks without relying on voltages, as the concept of power is handled directly through wave amplitudes and reflection coefficients.

Note that when maximum power transfer occurs (i.e., when $P_L = P_{av}$), the same amount of power is lost within the source itself. This level of inefficiency is unacceptable in high-power applications, such as those using magnetrons.

To express the internal power loss (10) in terms of the available power:

$$P_S = 4P_{av}\frac{R_S^2}{|Z_L + Z_S|^2} \qquad (14)$$

Likewise, the power (11) absorbed by the load becomes

$$P_L = 4P_{av}\frac{R_L R_S}{|Z_L + Z_S|^2} \qquad (15)$$

[4] A passive load is a load whose impedance has a non-negative real part, or equivalently, a reflection coefficient magnitude is less than or equal to 1.

2.3.4 Power to Match

The Power to Match (P_m) is the power a source would deliver to a perfectly matched load – that is, a load with zero reflection coefficient. This power can be expressed as the product of the source's available power and its mismatch factor:

$$P_m = P_{av}\left(1-\left|\Gamma_S\right|^2\right) = P_{av} m_S \qquad (16)$$

The quantity P_m is physically measurable and has a clear practical interpretation. In contrast, the available power P_{av} may not be physically meaningful in the context of high-power sources such as magnetrons. For such systems, it is often more appropriate to consider power to match P_m as the primary source characteristic, rather than P_{av}.

2.3.5 About Magnetron

"The magnetron is a strange beast."[5]

Unfortunately, the magnetron is far from a simple linear signal source. Its key parameters (internal impedance, generated power and oscillation frequency) are interrelated and are influenced by many factors, including anode current, applied DC magnetic field, filament current, and even the load reflection coefficient.

Nevertheless, despite this complexity, for the purposes of the following analysis, we will treat the magnetron as an idealized, linear signal source. This approximation is particularly justifiable when the magnetron is used in combination with a circulator and we are not concerned with the internal physical processes. A more detailed discussion of magnetron behavior will follow in Section Magnetron.

3. System Behavior

This section describes the dynamic processes that occur within the waveguide connecting the source to the load, culminating in the establishment of the *steady-state* situation. In practical industrial systems, steady-state is typically reached within tens of nanoseconds after an abrupt change (e.g., source turn-on). Although network theory, signal flow graphs, and scattering parameters provide powerful analytical tools, it is also instructive to consider the step-by-step buildup of the wave field in terms of successively arising incident and reflected waves.

To facilitate further discussion, the following conventions will be used:

- ***b*** denotes amplitudes of forward (incident) waves traveling from source to load.
- ***a*** denotes amplitudes of reverse (reflected) waves traveling from load to source.
- Subscripts ***S*** and ***L*** indicate positions at the source-waveguide and waveguide-load interface planes, respectively.
- A subscript ***n*** denotes the order of the wave (explained below).

Suppose the source is abruptly switched on. This generates a wave travelling toward the load, referred to as the *primary wave* or 1st-order incident wave, with complex amplitude b_{S1} at the source output plane. The power carried by this wave is equal to the power to match P_m, i.e., the power the source would deliver to a matched load:

[5] Interview with Robert L. Kyhl, conducted by William Aspray, 13 June 1991, http://ethw.org/Oral-History:Robert_Kyhl.

$$P_m = \frac{1}{2}|b_{S1}|^2 = P_{av} m_S$$

Assuming, without loss of generality, that b_{S1} is real, we obtain

$$b_{S1} = \sqrt{2P_m} = \sqrt{2P_{av} m_S} \tag{17}$$

As the wave propagates through the lossless waveguide, it reaches the load with the same magnitude after travelling the electric distance θ:

$$b_{L1} = b_{S1} \exp(-j\theta)$$

At the load, depending on the load reflection coefficient Γ_L, part of the wave is absorbed, and part is reflected, generating the 1st-order reflected wave, so named because its amplitude is proportional to the 1st power of the load reflection coefficient[6]:

$$a_{L1} = b_{L1}\Gamma_L = b_{S1}\Gamma_L \exp(-j\theta)$$

The reflected wave travels back toward the source, where it becomes

$$a_{S1} = a_{L1} \exp(-j\theta) = b_{S1}\Gamma_L \exp(-j2\theta)$$

At the source, which has a reflection coefficient Γ_S, the wave is partially absorbed and partially re-reflected, generating an additional wave travelling toward the load: the 2nd-order incident wave. The complex amplitude of this wave at the source and the load is, respectively

$$b_{S2} = a_{S1}\Gamma_S = b_{S1}\Gamma_S\Gamma_L \exp(-j2\theta)$$

$$b_{L2} = b_{S2} \exp(-j\theta) = b_{S1}\Gamma_S\Gamma_L \exp(-j3\theta)$$

The 2nd-order incident wave behaves in the same manner as the primary wave, generating a 2nd-order reflected wave. This reflected wave, in turn, gives rise to a 3rd-order incident wave. The process theoretically continues indefinitely (over an infinite amount of time), eventually leading to a steady state. However, in practice, only a finite number of higher-order contributions are significant, and a practically acceptable steady state is reached within a finite time – typically within the tens of nanoseconds previously mentioned.

The nth-order incident and reflected waves at the load interface are

$$b_{Ln} = b_{S1}\Gamma_S^{n-1}\Gamma_L^{n-1} \exp\left[-j\theta(2n-1)\right], \quad n=1,\ 2,\ 3,\ \ldots \tag{18}$$

$$a_{Ln} = b_{Ln}\Gamma_L = b_{S1}\Gamma_S^{n-1}\Gamma_L^{n} \exp\left[-j\theta(2n-1)\right], \quad n=1,\ 2,\ 3,\ \ldots \tag{19}$$

After theoretically infinite number of such interactions, a steady-state is established, as illustrated in Fig. 4. The total wave field is composed of an infinite series of terms with progressively decreasing absolute values. The total incident wave at the load is

$$b_L = \sum_{n=1}^{\infty} b_{Ln} = b_{S1}\sum_{n=1}^{\infty}\Gamma_S^{n-1}\Gamma_L^{n-1} \exp\left[-j\theta(2n-1)\right] \tag{20}$$

The total reflected wave at the load is

[6] A reflected wave is defined to be of order n if its complex amplitude is proportional to $(\Gamma_L)^n$, i.e., to the load reflection coefficient Γ_L raised to the nth power. An incident wave is said to be of order n if it gives rise to a reflected wave of the same order. Therefore, incident and reflected waves of the same order are always coupled through the load reflection coefficient Γ_L.

$$a_L = \sum_{n=1}^{\infty} a_{Ln} = \Gamma_L \sum_{n=1}^{\infty} b_{Ln} = \Gamma_L b_L$$

The complex ratio of the total wave amplitudes

$$a_L / b_L = \Gamma_L$$

remains equal to the load reflection coefficient, just as it does for each individual order n.

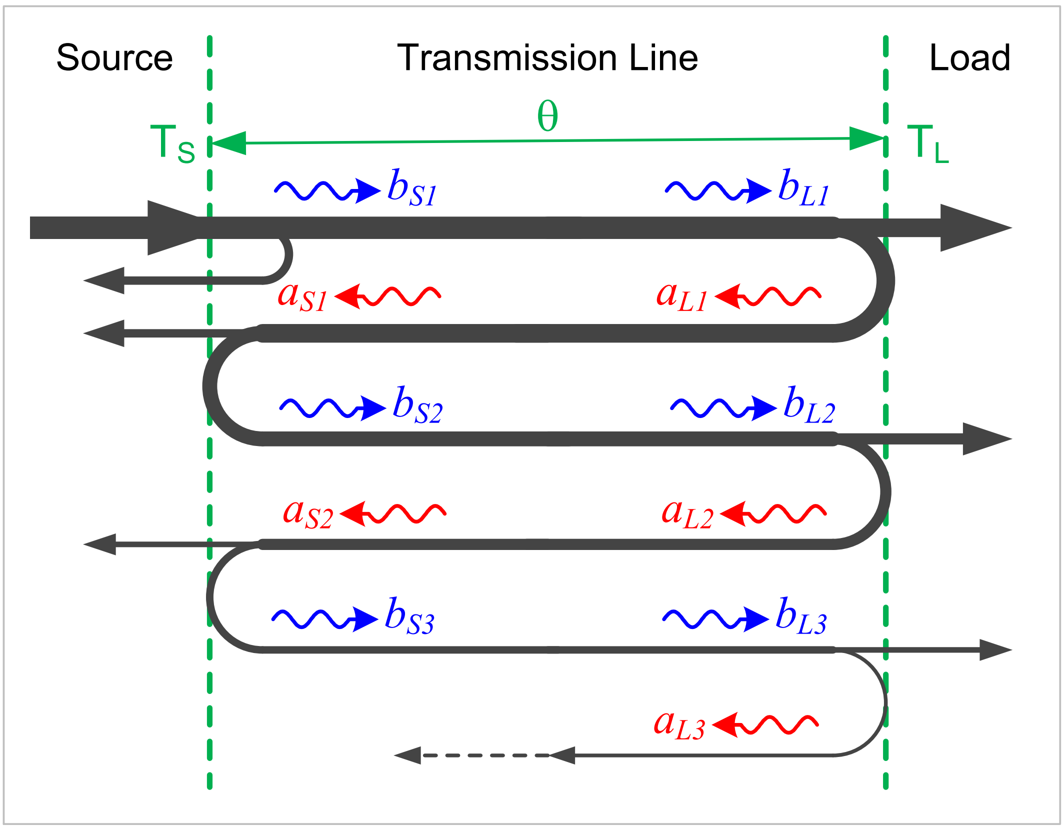


Fig. 4. Multiple reflections in a doubly-mismatched transmission line.

The infinite series (20) can be easily summed because it represents a geometric series with common ratio $q = \Gamma_S \Gamma_L \exp(-j2\theta)$, as we will prove now. Indeed, after substituting $k = n - 1$, the series becomes

$$b_L = b_{S1} \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_L^k \exp(-j\theta) \exp(-j2\theta k) = b_{S1} \exp(-j\theta) \sum_{k=0}^{\infty} \left[\Gamma_S \Gamma_L \exp(-j2\theta)\right]^k$$

Recalling that for $|q| < 1$

$$\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$$

we obtain

$$b_L = b_{S1} \frac{\exp(-j\theta)}{1 - \Gamma_S \Gamma_L \exp(-j2\theta)}$$

Using the definition (17) of b_{S1} in terms of the source available power, the final steady-state wave amplitudes at the load interface are for the incident and reflected wave, respectively:

$$b_L = \sqrt{2P_{av}m_S}\,\frac{\exp(-j\theta)}{1-\Gamma_S\Gamma_L\exp(-j2\theta)} \tag{21}$$

$$a_L = \Gamma_L b_L \tag{22}$$

These are the final formulas we wanted to arrive at.

4. Power Transfer

This section discusses key aspects of power transmission in the system, with a focus on the power absorbed by the load and the power incident upon it.

The power carried by the incident wave b_L (incident power or forward power) is

$$P_i = \frac{1}{2}|b_L|^2 = P_{av}\frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} = \frac{P_m}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{23}$$

where P_{av} is the available power of the source, and P_m is the power to match. The incident power can be measured using a directional coupler installed in the waveguide, the coupler oriented to detect the wave traveling toward the load.

The power carried by the reflected wave a_L (reflected power or reverse power) is

$$P_r = \frac{1}{2}|a_L|^2 = P_i|\Gamma_L|^2$$

This power can also be sampled using a directional coupler, but oriented to detect the wave traveling back toward the source.

In the following sections, we will explore the implications of these formulas and how they affect system behavior, particularly in cases of mismatched conditions.

4.1 Power Absorbed in Load

In high-power applications, the power absorbed in the load, also known as the *accepted power*, is of prime importance. By the principle of energy conservation, the absorbed power is simply the difference between the incident power and the reflected power:

$$P_L = P_i - P_r = P_i\left(1-|\Gamma_L|^2\right) = P_{av}\frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{24}$$

Equation (24) can be rewritten as

$$P_L = gP_{av}$$

where

$$g = \frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{25}$$

is the power transmission coefficient, also known as **transducer gain**. Transducer gain is often expressed in decibels (dB) as

$$G = 10\log(g)$$

The transducer gain relates the power actually absorbed in a load to the available source power. It is therefore a quantity of prime interest in the linear networks theory.

Transducer gain from (25) can be factored as

$$g = m_S\, h\, m_L \qquad (26)$$

where

$$m_S = 1 - |\Gamma_S|^2$$

$$m_L = 1 - |\Gamma_L|^2$$

$$h = \frac{1}{\left|1 - \Gamma_S \Gamma_L \exp(-j2\theta)\right|^2} \qquad (27)$$

Here m_S and m_L are the familiar source and load *mismatch factors*; we refer to h as the *interference factor*.

The source mismatch factor m_S is the fraction of the source available power that couples to a matched load[7]. Indeed, as we established in equation (16), if $\Gamma_L = 0$, then $g = m_S$.

The load mismatch factor is the fraction of available power from a *matched* source delivered to an arbitrary load. Like the source mismatch factor, if $\Gamma_S = 0$, then $g = m_L$.

It may be tempting to assume that when both the source and the load are mismatched, the transducer gain is simply the product of the two mismatch factors: a fraction m_S of the available power couples into the transmission line, and of that, only a fraction m_L reaches the load. However, this is not the case. When mismatches exist at both ends, multiple reflections occur between the source and the load—as previously discussed—which alter the overall power transmission. This interaction is precisely what the interference factor h accounts for.

Both the energy conservation principle and the analysis of (25) imply that for passive loads ($|\Gamma_L| \le 1$) the power absorbed in a load can never be greater than the power available from the source (i.e., $g \le 1$). In contrast, the power of an incident wave can be even substantially higher, as will be proved later.

As shown in Equation (27), the interference factor depends not only on the magnitudes of Γ_S and Γ_L, but also on their phases, as well as the electric length θ of the interconnecting waveguide. This is due to the term in the denominator

$$d = \Gamma_S \Gamma_L \exp(-j2\theta) = |\Gamma_S \Gamma_L| \exp\left[j(\varphi_S + \varphi_L - 2\theta)\right] = |\Gamma_S \Gamma_L| \exp(j\varphi_d)$$

In real systems, the phase angles φ_S, φ_L, θ are typically unknown and variable. Therefore, it is essential to understand the bounds of h over all possible phase combinations. Using these primary bounds, one can derive limits for all related quantities, such as transducer gain, power absorbed by the load, and incident power.

[7] It should be stressed that the *match* is related to the reference impedance Z_0, i.e., to the characteristic impedance of the interconnecting transmission line or waveguide.

The minimum value of the interference factor h occurs when the denominator in (27) is maximized, i.e., when $d = +|\Gamma_S \Gamma_L|$. Similarly, the maximum value of h occurs when the denominator is minimized, i.e., when $d = -|\Gamma_S \Gamma_L|$. Then

$$h_{\min} = \frac{1}{\left(1+\left|\Gamma_S \Gamma_L\right|\right)^2}, \qquad h_{\max} = \frac{1}{\left(1-\left|\Gamma_S \Gamma_L\right|\right)^2}$$

When at least one end of the transmission line is matched (i.e., $\Gamma_S = 0$ or $\Gamma_L = 0$), the interference factor becomes 1. As mismatch increases (i.e., as $|\Gamma_S \Gamma_L| \to 1$), the range of possible values widens to $h_{\min} \to 0.25$ and $h_{\max} \to +\infty$.

The bounds of the transducer gain are[8]

$$g_{\min,\max} = \frac{\left(1-\left|\Gamma_S\right|^2\right)\left(1-\left|\Gamma_L\right|^2\right)}{\left(1 \pm \left|\Gamma_S \Gamma_L\right|\right)^2} = m_S h_{\min,\max} m_L$$

The transducer gain can never exceed unity, and hence, the power transfer conforms to the energy conservation principle. We can also derive practical implications for the following special cases:

- If both transmission line ends are matched, then $g = 1$, and the available power is completely transmitted to load.
- If one transmission line end is matched, the transducer gain is equal to the mismatch factor of the other end, independent of phases.
- If the mismatches are equal ($|\Gamma_L| = |\Gamma_S|$), then $g_{\max} = 1$. This implies that by a correct phasing (e.g., via adjusting the waveguide length), total power transfer can be achieved.
- If the mismatches are unequal ($|\Gamma_L| \neq |\Gamma_S|$), then $g_{\max} < 1$. Total power transfer cannot be achieved by phasing alone.

In systems where available power is a theoretical extrapolation rather than directly measurable quantity (e.g., high-power magnetron-based sources), it is more meaningful to express absorbed power in terms of the power to match P_m. Equation (24) then reduces to

$$P_L = P_m \frac{1-\left|\Gamma_L\right|^2}{\left|1-\Gamma_S \Gamma_L \exp\left(-j2\theta\right)\right|^2} = P_m h m_L \qquad (28)$$

4.2 Incident Power

Equation (23) shows that incident power, the power carried by the forward-propagating wave b_L, depends not only on the source and load mismatch factors, but also on the interference factor h:

$$P_i = P_{av} \frac{1-\left|\Gamma_S\right|^2}{\left|1-\Gamma_S \Gamma_L \exp\left(-j2\theta\right)\right|^2} = P_{av} m_S h = P_m h \qquad (29)$$

This reveals a counterintuitive but important result: depending on the phase θ, the incident power P_i can actually exceed the gross power generated by the source! This occurs when multiple reflections, as explained in Section System Behavior, constructively interfere, reinforcing the field in the forward direction. However, it is also important to remember that the power absorbed in the load is determined by the difference between incident and reflected powers, and this net absorbed power P_L can never exceed the source available power P_{av}.

[8] The value of $g_{\max}$ becomes undefined when both $|\Gamma_S|$ and $|\Gamma_L|$ are equal to 1.

The maximum and minimum values of the incident power, relative to both available power P_{av} and the power to match P_m, are:

$$P_{i\,\min,\max} = P_{av}\frac{1-|\Gamma_S|^2}{\left(1 \pm |\Gamma_S \Gamma_L|\right)^2} = P_{av} m_S h_{\min,\max} = P_m h_{\min,\max} \tag{30}$$

Inspection of this formula shows that the upper bound of incident power exceeds the available power when

$$|\Gamma_L| > \frac{1}{|\Gamma_S|}\left(1-\sqrt{1-|\Gamma_S|^2}\right) \tag{31}$$

In terms of power to match, $P_{i\,\max} > P_m$ whenever both $|\Gamma_S| > 0$ and $|\Gamma_L| > 0$.

Fig. 5 illustrates this effect for a source with reflection coefficient $|\Gamma_S| = 0.25$.

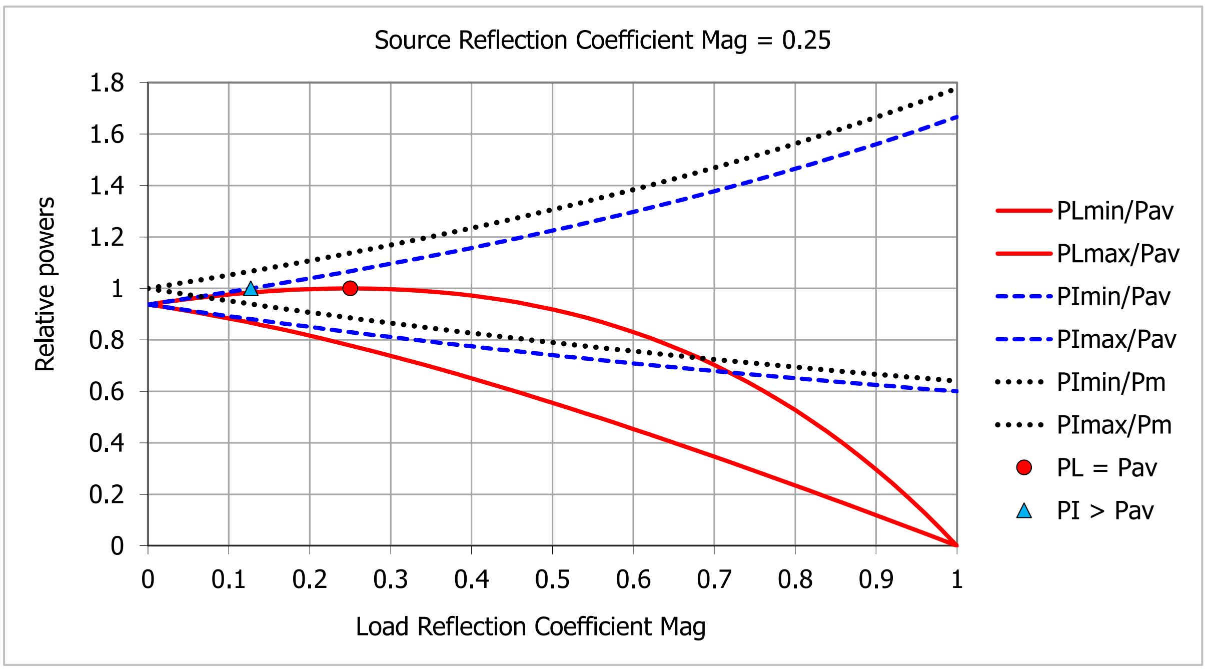


Fig. 5. Power ratios in a mismatched system as a function of load reflection coefficient magnitude $|\Gamma_L|$.

The red curves show the bounds of P_L/P_{av} as a function of $|\Gamma_L|$. The upper bound never exceeds unity and reaches exactly 1 when $|\Gamma_L| = |\Gamma_S|$. This condition, which allows ideal matching through an appropriate waveguide length (phase θ), is marked by the red circle.

The dashed blue curves represent the bounds of P_i/P_{av}. The ratio exceeds 1 when $|\Gamma_L| > 0.127$, consistent with Equation (31). This condition is indicated by the triangular marker.

The dotted black curves show the bounds of P_i/P_m. This ratio can exceed 1 depending on the phase θ, even in cases of slight mismatch.

Realistic Example

Suppose a circulator provides a source reflection coefficient of $|\Gamma_S| = 0.1$ (corresponding to 20 dB return loss). Then for an applicator with $|\Gamma_L| = 0.6$ (which is quite a likely case), the range of possible P_i/P_m is -0.89 to 1.13, which is a difference between -11% and 13%. With a better circulator with

$|\Gamma_S|$ = -25 dB, this interval reduces to -6% to 7%. For a 10-kW magnetron, this means -600 W to +700 W!

Simulations like these can be performed using the accompanying **PowTrans.exe** application.

Key Conclusions and Practical Considerations

- For given source and load reflection coefficients, the incident power P_i can be either lower or higher than the available power from the source P_{av}, depending on the length (and thus phase shift) of the interconnecting waveguide. In cases of significant mismatch, P_i can substantially exceed P_{av}. A similar relationship holds between the incident power P_i and the source's power to match P_m.
- As such, relying solely on P_i as an indicator of power delivered to the load can be misleading. Measuring only the incident power—commonly done using a single directional coupler—can result in considerable overestimation or underestimation of the actual power absorbed by the load, especially in systems where neither the source nor the load is well matched. This scenario is typical in magnetron-based setups without circulators.
- To accurately determine the power absorbed in the load, two directional couplers should be used: one to measure the incident power and another to measure the reflected power.
- In systems with high mismatch at both ends, the internal field strength within the waveguide can be very high, even when the net power transfer to the load is low. This elevated field strength can lead to localized heating, electrical breakdown, or arcing. Components such as tuning stubs, if present in such regions, may intensify or even initiate these effects.
- While incident power is not a reliable indicator of power absorbed by the load, it can serve as a useful measure of the internal electromagnetic field strength within the waveguide.
- The condition where the incident power exceeds the available source power does not violate the principle of energy conservation. This occurs because a corresponding portion of the incident power is reflected back toward the source. Assuming negligible transmission losses, the source ultimately delivers only the amount of power that is absorbed by the load.

5. Magnetron

The theoretical framework developed in the previous sections applies well to magnetron generators equipped with circulators. In such systems, the source is well matched and behaves in a largely linear and predictable manner.

However, the situation changes considerably when magnetrons are used without circulators, a common configuration in many applications, such as domestic microwave ovens.

Magnetrons are typically designed to operate optimally (i.e., to deliver maximum power to match P_m) under the following conditions:

- They are installed in a *reference launcher*, and
- The launcher output is terminated in a matched load.

A reference launcher is a coupling structure defined by magnetron manufacturers, serving as the basis for specifying the magnetron's performance characteristics—such as output power and frequency—in datasheets. The output of the launcher is typically a waveguide. Certain types of reference launchers are widely adopted and accepted across the industry.

An example of a magnetron installed in a reference launcher is shown in Fig. 6.

Datasheets for magnetrons generally include a *Rieke diagram*, which visualizes how the power P_L absorbed in load and oscillation frequency f_g depend on the load reflection coefficient Γ_L. This "Rieke" reflection coefficient can be referenced to the launcher output (as in Fig. 6), making it a physically measurable quantity[9]. In this context, a “match” implies that the Rieke reflection coefficient Γ_L, as defined, is zero.

For optimal performance, applicator designers must ensure that the input reflection coefficient of the applicator is as close to zero as possible, effectively fulfilling the same design goal as for systems that use circulators.

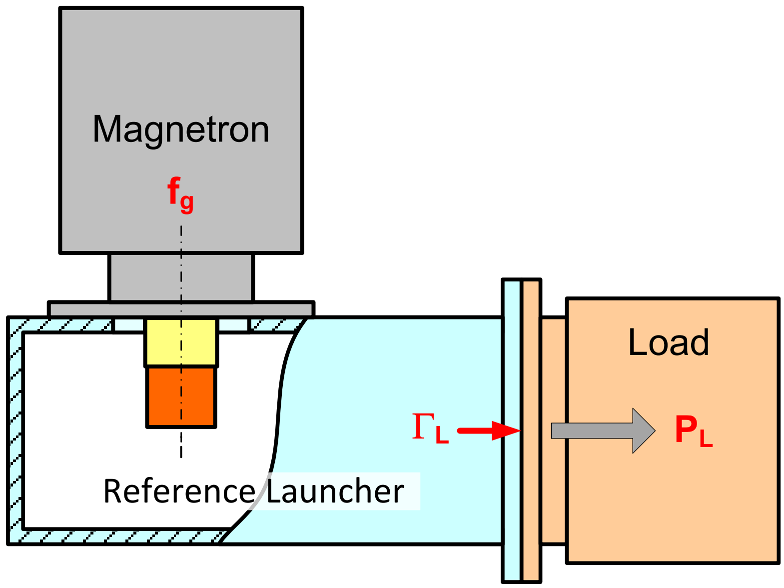


Fig. 6. Magnetron installed in a reference launcher and the definition of the “Rieke” reflection coefficient Γ_L.

In a magnetron generator that includes a circulator, the magnetron sees a nearly perfect match, and thus delivers the power to match P_m, which is equal to the incident power P_i, regardless of the load impedance Γ_L.

In systems without circulators, the situation is different. The incident power P_i depends strongly on the load reflection coefficient Γ_L. Therefore, using a dual directional coupler for power monitoring is essential. Relying on a single directional coupler to monitor only the incident power can result in misleading results.

Experimental Example

Fig. 7 shows measured power behavior for 2M244 magnetron with nominal frequency 2460 MHz and nominal power $P_m = 1.01$ kW. The powers were measured for a set of 661 load reflection coefficients shown, starting from the center of the polar diagram and “spiraling” outward (Fig. 7 left). Measured powers are plotted in Fig. 7 right.

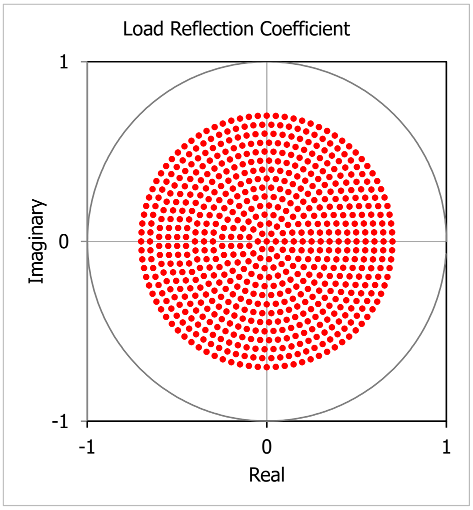
The green dashed horizontal line is the power to match (P_m), the result of the first measurement point, with nominal $\Gamma_L = 0$.

The red trace is the power P_L absorbed in the load. The ripple period corresponds to one “turn” of the load reflection coefficient Γ_L. As seen, P_L is always lower than P_m.

The black trace is the incident power P_i; the blue trace is the reflected power P_r. For some loads, the incident power is significantly higher than any actual power the magnetron could generate. This confirms that measuring only P_i is not sufficient. Instead, both the incident and reflected powers must

[9] Typically, though, the Rieke reflection coefficient is defined at the plane of the magnetron antenna axis.

be measured to compute the only relevant quantity, the power absorbed in the load, as the difference $P_L = P_i - P_r$. This is how the red trace in Fig. 7 was computed.



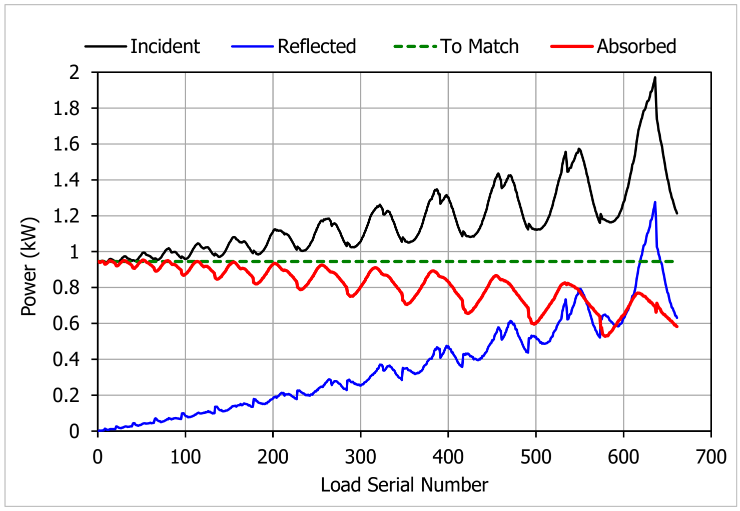


Fig. 7. Magnetron load reflection coefficients (left) and the corresponding measured powers (right).

6. What Should Be Measured

Building on the results presented in the previous sections, this section outlines which physical quantities should be measured to effectively characterize the system's behavior, how these measurements can be conducted, and what the limits of measurement accuracy are.

In high-power microwave systems, the key quantities of interest are:

- The power P_L absorbed by the load.
- The magnitude $|\Gamma_L|$ of the load reflection coefficient[10].
- The power P_r reflected from the load.

The absorbed power P_L is important not only for using energy as efficiently as possible, but also for controlling the processes within the working space.

The load reflection coefficient and the reflected power P_r are critical for protecting components such as the magnetron and circulator.

The quantities typically measured in practice are:

- Incident power P_i
- Reflected power P_r

From these, the power absorbed in the load is calculated as

$$P_L = P_i - P_r \tag{32}$$

and the magnitude of the load reflection coefficient as

$$|\Gamma_L| = \sqrt{P_r / P_i} \tag{33}$$

These powers are measured using directional couplers placed in the waveguide, oriented to sample the forward and reverse traveling waves, respectively.

[10] In the context of this chapter, the "load" is defined as perceived from the generator, consistent with the equivalent network Fig. 3. Therefore, Γ_L refers to the reflection coefficient Γ_{L1} as used in Fig. 1 and Fig. 2.

It is not uncommon for industrial systems to include only a forward-directional coupler, which can lead to the measured incident power P_i being mistakenly interpreted as the load power P_L . This can result in a paradoxical situation where the calculated load power appears to exceed even the DC input power to the magnetron. Such discrepancies frequently occur when magnetrons operate without circulators. The correct approach is to measure both the incident and reflected powers, and to compute their difference in order to accurately determine the power absorbed by the load.

Notes

- The single-coupler method is only justifiable when both the source and load are well matched (i.e., $\Gamma_S \approx 0$, $\Gamma_L \approx 0$).
- For higher accuracy and more comprehensive data, vector-based measurement techniques—such as the six-port reflectometer—are preferred. A vector reflectometer can simultaneously measure both the *complex* load reflection coefficient Γ_L and the incident power P_i . Moreover, access to the complex reflection coefficient allows for the correction of systematic measurement errors that significantly limit accuracy when relying solely on scalar (power-only) data from directional couplers. As an added benefit, knowledge of the complex reflection coefficient is essential for implementing effective automatic impedance matching.

6.1 Directional Couplers

In this section, we derive expressions for the coupled waves in directional couplers, which form the foundation for subsequent analysis of power and reflection coefficient measurements, as well as the estimation of measurement accuracy.

In high-power microwave systems, directional couplers typically feature very weak coupling (typically below -40 dB). As a result, they introduce negligible reflection into the main transmission line and can be considered perfectly matched at all ports. For this analysis, a directional coupler is characterized by two key parameters, both generally complex:

- Coupling factor c
- Directivity factor d

These quantities are often expressed in decibels (dB) as

$$C = 20\log|c|$$

$$D = -20\log|d|$$

A coupler is said to be perfectly directive when $d = 0$, which implies infinite directivity in dB.

We distinguish between two types of directional couplers, based on installation orientation.

- A *forward coupler* samples the incident wave b_L traveling toward the load. Its coupling and directivity factors are denoted c_i and d_i , respectively.
- A *reverse coupler* samples the reflected wave a_L traveling back toward the source. Its coupling and directivity factors are denoted c_r and d_r , respectively.

The signal emerging from the coupled port of the forward coupler is

$$b_{ci} = b_L c_i + a_L c_i d_i = b_L c_i (1 + \Gamma_L d_i) \qquad (34)$$

The signal emerging from the coupled port of the reverse coupler is

$$b_{cr} = a_L c_r + b_L c_r d_r = b_L c_r (\Gamma_L + d_r) \qquad (35)$$

These formulas describe how both the desired and undesired waves contribute to the measured signal as a function of the coupler's directivity. This "contamination" of the desired signals introduces error into calculations of both incident and reflected powers as well as reflection coefficient magnitude. The formulas enable us to estimate the measurement errors introduced by the finite directivity of the couplers.

If the couplers are ideally directive, i.e., $d_i = 0$, $d_r = 0$, these formulas simplify to

$$b_{ci} = b_L c_i \tag{36}$$

$$b_{cr} = b_L c_r \Gamma_L \tag{37}$$

In this ideal case, each coupler samples only the wave it is intended to measure, without contamination from the wave traveling in the opposite direction.

Another source of measurement errors stems from inaccurate knowledge of the coupling factors. The actual coupling factors c_i, c_r may differ from their *nominal* (specified) values c_{in}, c_{rn}. This discrepancy can lead to systematic measurement errors particularly when trying to calculate absolute power levels.

For precise analysis, it is essential to account for both finite directivity and inaccuracies in coupling factor estimates.

6.2 Power Measurement

The power carried by a wave emerging from the coupled port of a directional coupler is proportional to the squared magnitude of the coupled wave:

$$P_c = \frac{1}{2}|b_c|^2$$

Applying this to the coupled sample b_{ci} of the incident wave, and to the coupled sample b_{cr} of the reflected wave, we obtain

$$P_{ci} = P_i |c_i|^2 |1 + \Gamma_L d_i|^2$$

$$P_{cr} = P_i |c_r|^2 |\Gamma_L + d_r|^2$$

where

$$P_i = \frac{1}{2}|b_L|^2$$

is the power incident on the load. The power reflected from the load is $P_r = P_i |\Gamma_L|^2$.

The powers P_{ci} and P_{cr} are what is actually measured by power meters connected to the forward and reverse coupled ports, respectively. For this analysis, we assume the coupled powers are measured accurately.

With perfect directional couplers ($d_i = 0$, $d_r = 0$), P_{ci} is proportional to the incident power P_i and P_{cr} is proportional to the reflected power P_r:

$$P_{ci} = |c_i|^2 P_i$$

$$P_{cr} = |c_r|^2 P_i |\Gamma_L|^2 = |c_r|^2 P_r$$

Therefore, to compute the actual incident and reflected powers from the measured values, the coupled powers P_{ci} and P_{cr} must be divided by $|c_i|^2$ and $|c_r|^2$, respectively. However, the true values of the coupling factors c_i, c_r are often not accurately known. Instead, measurements are made using the nominal values c_{in}, c_{rn}. This leads to the measured power estimates:

$$P_{mi} = \frac{P_{ci}}{|c_{in}|^2} = P_i \left|\frac{c_i}{c_{in}}\right|^2 |1 + \Gamma_L d_i|^2 \qquad (38)$$

$$P_{mr} = \frac{P_{cr}}{|c_{rn}|^2} = P_i \left|\frac{c_r}{c_{rn}}\right|^2 |\Gamma_L + d_r|^2 \qquad (39)$$

As can be seen, the *measured* values P_{mi} and P_{mr} generally differ from the true powers P_i and P_r due to two sources of measurement errors:

- Finite directivity of couplers.
- Inaccurate knowledge of coupling factors.

These two error sources are the primary causes of inaccuracy when using directional couplers for microwave power and reflection coefficient measurements. The impact of these error sources will be examined in the following sections.

6.2.1 Finite Directivity Error

To isolate the effects of finite directivity, let us assume the coupling factors are exactly known, i.e., $c_{in} = c_i$, $c_{rn} = c_r$. Under this assumption, Equations (38), (39) simplify to

$$P_{mi} = P_i |1 + \Gamma_L d_i|^2 \qquad (40)$$

$$P_{mr} = P_i |\Gamma_L + d_r|^2 \qquad (41)$$

These measured values depend not only on the magnitude $|\Gamma_L|$, but also on the phase of the load reflection coefficient, as well as the phase and magnitude of the directivity factors d_i and d_r . Since the phases are typically unknown, the uncertainty in power measurement must be expressed using minimum and maximum bounds over all possible phase combinations.

We will distinguish between two cases based on what is known about the directivity:

- **Case Known**: The *exact* modulus $|d|$ of the directivity factor is known (for example, we know that that a coupler's directivity is 27 dB, and hence $|d| = 0.045$).
- **Case Max**: Only a *maximum* bound $d_{\max}$ of the modulus is guaranteed (for example, we know that the coupler's directivity is not lower than 25 dB, implying $d_{\max} = 0.056$). In this case the exact value of $|d|$ is unknown but can be anywhere within the range $0 \le |d| \le d_{\max}$.

Limits of Incident Power

Because $|\Gamma_L d_i| < 1$, the situation for $d_{\max}$, estimating the bounds of measured incident power P_{mi} is rather straightforward. The blue circle in Fig. 8 illustrates all possible complex values of $A = 1 + \Gamma_L d_i$.

For **Case Known**, the bounds of P_{mi} are

$$P_{mi\min} = P_i \left(1 - |\Gamma_L|\,|d_i|\right)^2$$

$$P_{mi\max} = P_i \left(1 + |\Gamma_L|\,|d_i|\right)^2$$

For **Case Max**, using $d_{i\max}$:

$$P_{mi\min} = P_i \left(1 - |\Gamma_L|\, d_{i\max}\right)^2$$

$$P_{mi\max} = P_i \left(1 + |\Gamma_L|\, d_{i\max}\right)^2$$

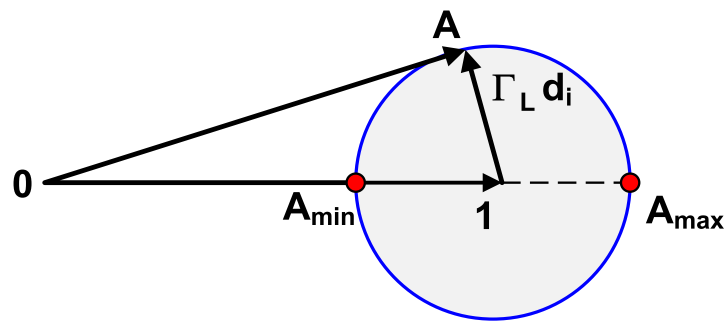


Fig. 8. Geometric illustration for determining bounds of the measured incident power.

Limits of Reflected Power

For **Case Known**, using a similar approach, the limits of P_{mr} are

$$P_{mr\min} = P_i\left(\left|\Gamma_L\right| - \left|d_r\right|\right)^2$$

$$P_{mr\max} = P_i\left(\left|\Gamma_L\right| + \left|d_r\right|\right)^2$$

Note: if $|\Gamma_L| = |d_r|$, the measured reflection coefficient could be zero, even when actual reflection exists.

For **Case Max**, we distinguish two sub-cases: $|\Gamma_L| > d_{r\,\max}$ and $|\Gamma_L| \le d_{r\,\max}$. Fig. 9 shows values of vector $A = \Gamma_L + d_r$ for all phase combinations of Γ_L and d_r. For a fixed Γ_L, the darker circle represents the region of all possible values of A for unknown phase of d_r and magnitude of d_r varying between 0 and $d_{r\,\max}$. The lighter-shaded area extends this by also varying the phase of Γ_L. The figure illustrates the case $|\Gamma_L| > d_{r\,\max}$.

In both sub-cases, the maximal measured reflected power is

$$P_{mr\max} = P_i\left(\left|\Gamma_L\right| + d_{r\max}\right)^2$$

If $|\Gamma_L| > d_{r\,\max}$, the minimal measured power is

$$P_{mr\min} = P_i\left(\left|\Gamma_L\right| - d_{r\max}\right)^2$$

However, when $|\Gamma_L| \le d_{r\,\max}$, the origin is within the uncertainty region, allowing $P_{mr\,\min} = 0$.

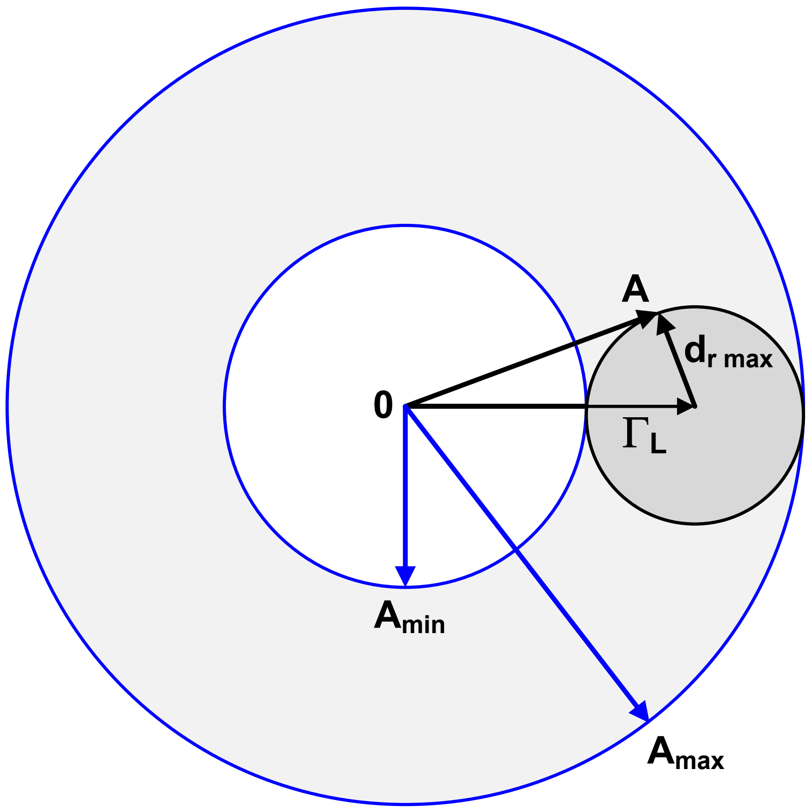


Fig. 9. Geometric illustration for determining bounds of the reflected power for the case of $|\Gamma_L| > d_{r\,max}$. In the case of $|\Gamma_L| \le d_{r\,max}$, the darker circle always comprises zero, and therefore $A_{min} = 0$.

6.2.2 Unknown Coupling Error

Now suppose the couplers are perfectly directive, i.e., $d_i = 0$, $d_r = 0$. In this case, Equations (38), (39) simplify to

$$P_{mi} = P_i \left|\frac{c_i}{c_{in}}\right|^2, \quad P_{mr} = P_r \left|\frac{c_r}{c_{rn}}\right|^2 \qquad (42)$$

Here, the only source of measurement error is the inaccurate knowledge of the coupling factors c_i and c_r.

The coupling factor uncertainty is usually expressed in decibels as $\Delta C \ge 0$. The actual coupling C (dB) is then bounded by

$$C_{\min} = C_n - \Delta C\,, \quad C_{\max} = C_n + \Delta C$$

where $C_n = 20 \log|c_n|$ is the nominal coupling factor in dB. If the measured power is also expressed in dB units (e.g., dBm), then ΔC directly represents the power measurement uncertainty in decibels.

In the linear scale, the bounds of the coupling factor become:

$$c_{\min} = |c_n| 10^{-\Delta C/20}\,, \quad c_{\max} = |c_n| 10^{+\Delta C/20}$$

This leads to the multiplicative error bounds on the measured power

$$\left|\frac{c_{\min}}{c_n}\right|^2 = 10^{-\Delta C/10}, \quad \left|\frac{c_{\max}}{c_n}\right|^2 = 10^{+\Delta C/10}$$

Using Equation (42), and applying the bounds to both forward and reverse couplers, we get

$$P_{mi\min} = P_i 10^{-\Delta C_i/10} = P_i / e$$

$$P_{mi\max} = P_i 10^{\Delta C_i/10} = P_i e$$

$$P_{mr\min} = P_r 10^{-\Delta C_r/10} = P_r / e$$

$$P_{mr\max} = P_r 10^{\Delta C_r/10} = P_r e$$

where

$$e = 10^{\Delta C_r/10}$$

In Table 1, we examine the practical implications of this.

Table 1. Power measurement error due to coupling factor uncertainty.

ΔC (dB)	0	0.1	0.2	0.5	1	2
e	1.000	1.023	1.047	1.122	1.259	1.585

This table shows how even small uncertainties in coupling factor can lead to non-negligible power reading errors, particularly in high-power applications. Suppose, for instance, that the coupling factor is known with an uncertainty of ±0.5 dB, which represents relatively good accuracy. Then a true power of 10 kW can be measured anywhere between 8.9 kW and 11.2 kW.

6.2.3 Total Error

The aggregate uncertainty in power measurement arises from two main sources: finite directivity of directional couplers and uncertainty in coupling factors. For **Case Known** (i.e., when both the magnitudes $|d_i|$, $|d_r|$ of the directivity factors and the coupling uncertainty ΔC are known, the total bounds on the measured incident and reflected powers are

$$P_{mi\min} = P_i \left(1 - |\Gamma_L|\,|d_i|\right)^2 10^{-\Delta C_i/10}$$

$$P_{mi\max} = P_i \left(1 + |\Gamma_L|\,|d_i|\right)^2 10^{\Delta C_i/10}$$

$$P_{mr\min} = P_i \left(|\Gamma_L| - |d_r|\right)^2 10^{-\Delta C_r/10}$$

$$P_{mr\max} = P_i \left(|\Gamma_L| + |d_r|\right)^2 10^{\Delta C_r/10}$$

For other uncertainty models (e.g., **Case Max**, where only bounds are known), the total error expressions follow analogously. Their detailed derivation is left to the reader.

6.2.4 Absorbed Power Uncertainty

The uncertainty bounds on the power P_L absorbed in the load are derived by evaluating the worst-case difference between the incident and reflected power measurements:

$$P_{L\min} = P_{mi\min} - P_{mr\max}$$

$$P_{L\max} = P_{mi\max} - P_{mr\min}$$

These formulas provide a conservative estimate of the possible range of actual absorbed power, accounting for all known uncertainties in coupler directivity, coupling factor, and reflection coefficient phase and magnitude.

6.3 Reflection Coefficient Measurement

Using the results from Section Directional Couplers for ideally-directive couplers ($d_i = d_r = 0$), the load reflection coefficient can be calculated from measured wave amplitudes as

$$\Gamma_L = \frac{b_{cr}}{b_{ci}} \frac{c_i}{c_r}$$

The ratio c_i / c_r accounts for unequal coupling factors in the forward and reverse couplers.

In practical systems, the true coupling factors are not precisely known, so nominal values c_{in}, c_{rn} are used instead. The resulting measured reflection coefficient is

$$\Gamma_m = \frac{b_{cr}}{b_{ci}} \frac{c_{in}}{c_{rn}} = \frac{c_r}{c_{rn}} \frac{c_{in}}{c_i} \frac{\Gamma_L + d_r}{1 + \Gamma_L d_i} \qquad (43)$$

The measured reflection coefficient Γ_m generally differs from the actual value Γ_L. Three sources of errors can be identified:

1. **Tracking error** is caused by unequal coupling factors of the forward and reverse couplers (causing the multiplicative factor to differ from unity). This affects equally reflection coefficients of all magnitudes. In scalar systems, this error can be partially corrected by scaling the measured $|\Gamma_m|$ using a short-circuit calibration[11].
2. **Directivity error** arises from the finite directivity of the reverse coupler (nonzero d_r). It is the dominant source of error when measuring small reflection coefficients. The formula indicates that, in the absence of the tracking error, a perfectly matched load ($\Gamma_L = 0$) would be incorrectly measured as $\Gamma_{Lm} = d_r$.
3. **Source mismatch error** results from the finite directivity of the forward coupler (nonzero d_i), i.e., its inability to accurately measure the incident power[12]. This error becomes most significant when measuring high reflection coefficients. The error becomes negligible for small reflection coefficients, as the denominator of (43) approaches unity in such cases.

In scalar measurement setups (which only detect powers), the measured reflection coefficient is calculated as

$$|\Gamma_m| = \sqrt{\frac{P_{mr}}{P_{mi}}}$$

The formula relies on power measurements, so the uncertainties in P_{mi} and P_{mr}, derived in Section Power Measurement, also affect $|\Gamma_m|$ and can be used for its uncertainty estimation. Alternatively, using Equation (43),

---

[11] In short-circuit calibration, a short circuit (or any other fully reflecting termination) is connected in place of the actual load, and the resulting measured reflection coefficient magnitude is recorded. All subsequent measurements are then normalized by dividing them by this recorded value.

[12] This effect is similar to the case of an ideal coupler (i.e., $d_i = 0$), but with a discontinuity between the coupler and the measured load. Re-reflections from this discontinuity modify the actual incident wave, but these changes are not captured by the coupler, since it is positioned closer to the generator. This introduces an effective source mismatch.

$$\left|\Gamma_m\right| = \left|\frac{c_r}{c_{rn}}\frac{c_{in}}{c_i}\right|\left|\frac{\Gamma_L + d_r}{1+\Gamma_L d_i}\right|$$

Assuming the **Case Known** conditions in Section Power Measurement (where coupling and directivity factors are known), and using the formulas derived in Section Total Error, we have

$$\left|\Gamma_m\right|_{\min} = \sqrt{\frac{P_{mr\min}}{P_{mi\max}}} = \frac{\left|\left|\Gamma_L\right| - \left|d_r\right|\right|}{1+\left|\Gamma_L\right|\left|d_i\right|}10^{-(\Delta C_i + \Delta C_r)/10}$$

$$\left|\Gamma_m\right|_{\max} = \sqrt{\frac{P_{mr\max}}{P_{mi\min}}} = \frac{\left|\Gamma_L\right| + \left|d_r\right|}{1-\left|\Gamma_L\right|\left|d_i\right|}10^{(\Delta C_i + \Delta C_r)/10}$$

Supposing that the coupling factors uncertainties were eliminated by a short-circuit calibration, the formulas simplify to

$$\left|\Gamma_m\right|_{\min} = \frac{\left|\left|\Gamma_L\right| - \left|d_r\right|\right|}{1+\left|\Gamma_L\right|\left|d_i\right|} \tag{44}$$

$$\left|\Gamma_m\right|_{\max} = \frac{\left|\Gamma_L\right| + \left|d_r\right|}{1-\left|\Gamma_L\right|\left|d_i\right|} \tag{45}$$

6.4 Estimates of True Values

The formulas derived in the earlier sections allow us to determine the range of possible measured values, assuming the true values (e.g., P_i, Γ_L) are known. However, in practice, we are usually faced with the inverse problem: given measured values, such as P_{mi} and $|\Gamma_m|$, what are the likely bounds within which the true values lie? While deriving exact inverse formulas is theoretically possible, a practical and sufficiently accurate alternative is to apply the original formulas in reverse. This approach is valid if the measurement uncertainties are moderate and the resulting bounds are reasonably narrow, ensuring meaningful estimation.

To estimate uncertainty intervals for the true values, we start with the same bounding formulas from the Sections Total Error and Reflection Coefficient Measurement. We then substitute the measured values P_{mi}, P_{mr}, and $|\Gamma_m|$ in place of the true values P_i, P_r, and $|\Gamma_L|$. This yields worst-case bounds on the true values. For example:

$$P_{i\min} = P_{mi}\left(1-\left|\Gamma_m\right|\left|d_i\right|\right)^2 10^{-\Delta C_i/10}$$

$$P_{i\max} = P_{mi}\left(1+\left|\Gamma_m\right|\left|d_i\right|\right)^2 10^{\Delta C_i/10}$$

$$\left|\Gamma_L\right|_{\min} = \frac{\left|\left|\Gamma_m\right| - \left|d_r\right|\right|}{1+\left|\Gamma_m\right|\left|d_i\right|}$$

$$\left|\Gamma_L\right|_{\max} = \frac{\left|\Gamma_m\right| + \left|d_r\right|}{1-\left|\Gamma_m\right|\left|d_i\right|}$$

These formulas are implemented in the accompanying **PowTrans.exe** application to estimate true power and reflection coefficient bounds.

Once bounds on the reflection coefficient $|\Gamma_L|$ are known, corresponding values for VSWR (S) and return loss (R) can be computed using standard transformation formulas:

$$S = \frac{1+|\Gamma_m|}{1-|\Gamma_m|}$$

$$S_{\min} = \frac{1+|\Gamma_L|_{\min}}{1-|\Gamma_L|_{\min}}$$

$$S_{\max} = \frac{1+|\Gamma_L|_{\max}}{1-|\Gamma_L|_{\max}}$$

$$R = -20\log|\Gamma_m|$$

$$R_{\min} = -20\log|\Gamma_L|_{\max}$$

$$R_{\max} = -20\log|\Gamma_L|_{\min}$$

7. Conclusions

- This article has analyzed a linear microwave system consisting of a signal source, a lossless transmission line (waveguide), and a load.
- It was shown that while the power absorbed by the load can never exceed the available power from the source, the power carried by traveling waves in the waveguide can, under certain conditions, exceed both the source's available power and the power delivered to a matched load. This effect arises in cases of simultaneous mismatch at both the source and load.
- The powers associated with these traveling waves depend not only on the magnitudes of the source and load reflection coefficients, but also on their relative phases and the electric length of the transmission line.
- Incident power alone is an unreliable indicator of the power absorbed by the load. Accurate determination of absorbed power requires measuring both incident and reflected power, with the net absorbed power given by their difference.
- Incident power is more accurately viewed as an indicator of the *field intensity* within the waveguide. In systems with significant mismatches at both ends, the structure can act like a loosely coupled resonator, producing strong internal fields even if little net power is transmitted, raising the risk of overheating or electrical breakdown.
- This study also examined the use of directional couplers for measuring power and reflection coefficient. It presented methods to calculate worst-case uncertainty bounds, account for finite directivity, and for coupling factor tolerances. These results provide guidelines for practical measurement setup design and error estimation.

To explore the theoretical insights quantitatively—or to simulate specific real-world configurations—users are encouraged to utilize the accompanying tool: Microwave Power Transmission Calculator (**PowTrans.exe**). This application enables users to:

- Model a signal source (e.g., a magnetron) and a load (e.g., applicator or working chamber).
- Simulate power flow between source and load, including impact of phase variations and mismatches.
- Estimate measurement uncertainties associated with directional coupler-based measurements of power and reflection coefficient.